Title: Stringing Along with Radians

Brief Overview:

This activity is designed to reinforce student knowledge of the properties of the circle and to introduce them to the concept of radian measure and its connection to the study of trigonometry. The lesson is divided into four parts. The first part is a hands-on activity using string, chalk, and protractor. The second part of the activity is a follow-up using graphics technology to model the initial activity and introduce the term radian. The third part contains practice problems which involve discovery and hypothesis of ways to convert degree measures into radians and radian measures into degrees. The fourth part allows students to explore applications involving radian measures.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems through the use of radian measure.

- **Reasoning** Students will learn the meaning of radians by physically transferring the measurement of the radius onto the circle. Students will also discover the relationship between radian and degree measurement and explore applications.

- **Number Relationships** Students will investigate the relationships between π and the radius and circumference. Students will discover that the radian measure of a circle is 2π.

Grade/Level:

Grades 9-12

Duration/Length:

This activity will take 1- 2 ninety-minute blocks, or it could be broken into 3 - 4 forty-five minute periods. The activities may take longer than anticipated depending on class duration and extent of exploration.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Measuring lengths in centimeters and measuring angles using a protractor
- Making simple constructions using Cabri Geometry on the TI-92
- Using basic vocabulary of the circle

Objectives:

Students will be able to:

- work cooperatively in groups.
- collect and organize data.
- model the activity using Cabri Geometry on the TI-92.
- discover and hypothesize ways to convert degree measures into radians and radian measures into degrees.
- solve problems using radian measurement.

Materials/Resources/Printed Materials:

- Paper
- Pencil
- Chalk
- Ruler
- Protractor
- Shoestring or a string of random length, one per cooperative group
- TI-92, 1 per cooperative group
- Blacktop or large carpeted room
- Student Worksheets 1, 2, 3a - 3c, 4
- Teacher Solutions

Development/Procedures:

- Complete Lab Activity #1.

- Complete Lab Activity #2.

- Complete Worksheet 3A: Radian Measure of Special Angles.

- Complete Worksheet 3B: Radian and Degree Connections.

- Complete Worksheet 3C: Radian and Degree Measure Beyond the Special Angles.

- Complete Worksheet 4: Real-Life Applications.

Evaluation:

- Given a protractor marked in degrees and angles to be measured, students will compute radian measures.

- Students will solve real-life application problems to demonstrate their understanding and knowledge of circular functions.

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LAB ACTIVITY #1

Complete the following steps. Collect and record your data as you work through the activity.

<u>MATERIALS FOR EACH COOPERATIVE GROUP OF 3-4 STUDENTS</u>

- One shoestring per cooperative group of 3-4 students
- Chalk
- Ruler (metric)
- Protractor
- Pencil

1. Draw a point on the carpet/blacktop with your chalk.
2. Have one student hold one end of the shoestring on the point drawn and another student hold the other end of the shoestring to a piece of chalk.
3. Hold the string taut and carefully draw a circle.
4. Mark a point on the circumference of the circle.
5. Draw a radius connecting the center point to the point on the circumference.
6. Place the shoe string on the circumference of the circle, placing one end of the string on the point on the circle. Mark another point on the circle at the other end of the string.
7. Continue mapping the length of the shoestring around the circumference of the circle making sure that you mark the length along the circumference as you go.
8. Draw segments from the center of the circle to each of the marks you made around the circumference of the circle.
9. Label each of the angles beginning with angle #1 and numbering consecutively until all angles are labeled.
10. Use the protractor to obtain the approximate angle measure for each central angle in your circle.
11. Complete the accompanying data sheet for this activity.

LAB ACTIVITY #1 - DATA SHEET

1. MEASUREMENT
a. Length of your shoestring. _____________
b. Approximate measure of each angle in the circle.
$\angle 1$= ______ $\angle 2$= ______ $\angle 3$= ______ $\angle 4$= ______ $\angle 5$= ______ $\angle 6$= ______ $\angle 7$= ______

2. MORE ABOUT THE ANGLES
a. How many of your angles are approximately the same in measure?_______
b. What is the sum of all the angles that are about the same measure?________
c. What is the difference between 360^{o} and the sum you found above? ________
d. What is the measure of the only angle not included in the sum above? ________
e. What should the sum of all the central angles you drew equal? ___________
f. What is the actual sum of all the angles you measured? ________ If your actual sum is different the answer you gave in Part 2e, what do think caused this to happen?________
____
____
g. If you were to cut the circle in half (having only 180^{o}), how many of these angles from part 2a can you draw within the half circle you now have? _____________________________
h. With your previous observation in mind, how many of these same angles could you then draw within the entire circle?_______________________

3. APPLYING WHAT YOU KNOW
a. Based on your knowledge of how the circle and its components were constructed, how might you give an approximate circumference of the circle? (Give an explanation that supports your response.)____
____
____

b. Estimate what you think the quotient would be if you divided the circumference of your circle by the length of your shoestring?______

4. EXTENDING YOUR THOUGHTS
a. How do you think this quotient would change if you used a longer/shorter shoestring? (Support your speculations using knowledge from this activity.) _____________________
____
____
____
____
____

LAB ACTIVITY #2 - DATA SHEET

THE TI-92

1. Turn on the TI-92.

2. Press applications APPS, choose 8: GEOMETRY. Choose 3: NEW, press enter.

3. Move the cursor to VARIABLE, type in LAB2, press enter twice. (You have now opened a file called “LAB2" to save the work you are about to do.)

4. Press F2, choose 1: Point, enter. You should see a pencil on the screen. Use the cursor pad to move the pencil around. When the cursor moves to the middle of the screen, press enter. You should now have a point drawn in the middle of the screen.

5. Now you want to represent your shoestring onto the screen, but your shoestring is way too long! So, we need to use a scale factor so we can represent our shoestring on the screen.

   A) The length of your shoestring is __________ cm.

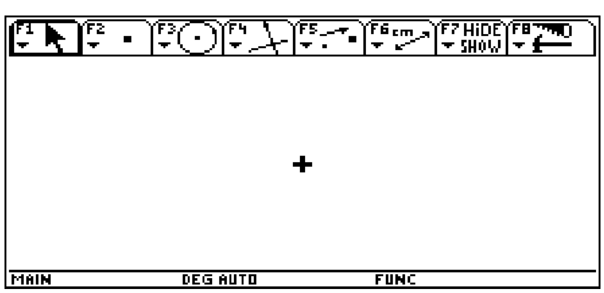
   B) The scale factor of the segment you want to construct on the TI-92 to the length of your shoestring is 1:50.

   C) Press F6, choose 6: Calculate, enter. The cursor should be blinking in the status bar. Enter the length of your shoestring, divide by 50, enter. R:_____ should appear on the screen. This number, _____, represents the length to scale which can be accommodated on your view screen.

6. Press F4, choose 9: Measurement Transfer, enter. Move the cursor to the scale number on the screen, press enter. Move the cursor to the point, press enter twice.

7. Press F2, choose 5: Segment, enter. Draw a segment connecting these two points. This segment will represent the radius of a circle.

8. Press F3, choose 1: Circle, press enter twice. Draw a circle with the center at the original point and the radius point at the other end of the segment. On this paper, copy what you see on the screen.



9. Press F4, choose 9: Measurement Transfer, enter. Move the cursor to the scale number on the screen, press enter. Move the cursor to the circle, press enter. Move the cursor to the radius point, press enter. Observe a new point plotted on the circle.

10. Repeat this measurement transfer enough times to map the circumference of the circle. Are all arc lengths equal? ___________ . Based on your experience with the shoestring, should you expect to see one smaller arc? ___________

11. Using the segment tool, draw the radii formed by the center point and each point on the circle.

12. Next, you will measure each central angle. Press F6, choose 3: Angle, enter. Move the cursor to any of the points constructed on the circle, press enter. Move the cursor to the center point, press enter. Move the cursor to a consecutive point constructed on the circle, press enter. The measure of each central angle in the circle is:

$\angle 1$ = _____ $\angle 2$ = _____ $\angle 3$ = _____ $\angle 4$ = _____ $\angle 5$ = _____ $\angle 6$ = _____ $\angle 7$ = _____

13. What is the sum of the measures of the angles in the circle? ___________

14. What did you expect the sum to be? ___________

15. Why is there a difference?

____

____

____

16. As a way of describing the angles, let us NOT measure angles in degrees anymore. Instead, we will measure them in terms of each of these congruent central angles. Since the radius was used to construct the arc length of each angle, we will refer to the measure of each central angle as **1 radian.**

17. How many degrees are in 1 radian? ___________

18. How many radians are in 1 degree? ___________

19. How many radians are in a circle? ____________

20. How many radians are in a semicircle? ___________

21. What is the significance of the answer to question 20?

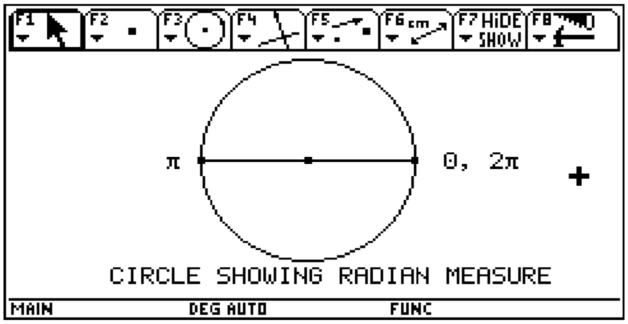
____

____

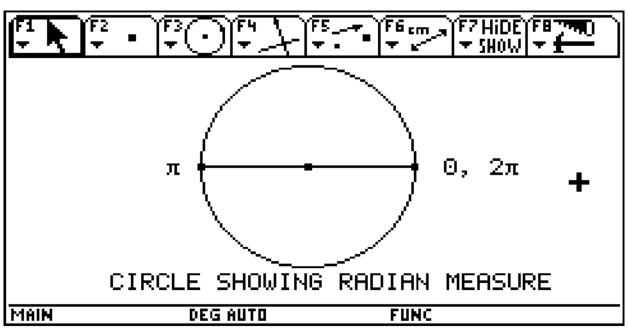
____

RADIAN MEASURE OF SPECIAL ANGLES: AN APPLICATION

- Given a representative sketch showing the circle, its diameter and 0, π, and 2π radians, explore the radian measure of special angles. (A new sketch is provided for each exploration so that you can mark your circle to illustrate your responses.)



1. If a half rotation (180°) is equal to π radians, what is the radian measure for 90°? ________
   What is the radian measure for 270°?____________



0° = ____________

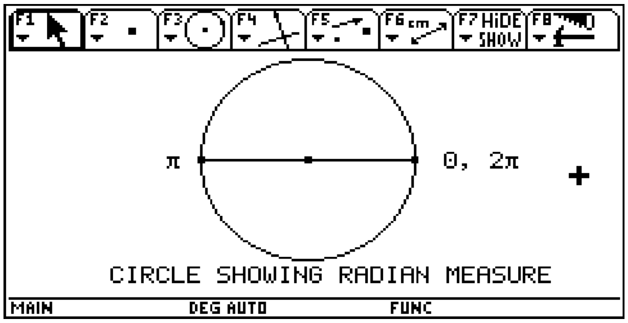
90° = ____________

180° =____________

270° = ____________

360° = ____________

2. Given the relationship of 180º and π radians, use the circle provided and label 45º, 135º, 225º, and 315º using radian measure.



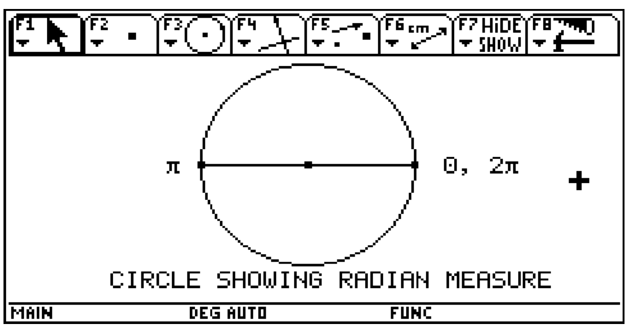
45° = ___________

135° = ___________

225° = ___________

315° = ___________

3. Using the same procedure as in #2, use the circle below and label 30°, 60°, 120°, 150°, 210°, 240°, 300°, and 330° using radian measure.



30° = ___________

60° = ___________

120° = ___________

150° = ___________

210° = ___________

240° = ___________

300° = ___________

330° = ___________

RADIAN & DEGREE CONNECTIONS

Given your knowledge of the radian measure of special angles, let's explore the connections between radian and degree measure. Based on your previous exploration, give the degree measure that is equal to each of the following radian measures.

$\frac{\pi}{4}$ =

$\frac{\pi}{6}$ =

$\frac{2\pi}{3}$ =

$\frac{3\pi}{4}$ =

$\frac{11\pi}{4}$ =

Based on the knowledge you now have of radian and degree measure, explain to a student who missed this lab experience how to convert from radian to degree measure and then from degree to radian measure.

To convert from radian to degree measure you should

____

____

____

To convert from degree to radian measure you should

____

____

____

____

Based on your previous two answers, is there a mathematical formula that supports the procedures you described? If so, what are they?

____

____

____

RADIAN AND DEGREE MEASURE BEYOND THE SPECIAL ANGLES

Now that you have made connections between radian and degree measure, and you have discovered how to convert between radians and degrees, you are ready to explore simple conversions for measures that are not special angles.

Using the formulas from the previous exercise, convert each radian measure into degrees, and each degree measure into radians.

Use your calculator in both radian and degree mode to verify your solutions.

$\frac{5\pi}{2} =$

$115° =$

$\frac{7\pi}{12} =$

$155° =$

$\frac{3\pi}{18}$

$310° =$

$\frac{4\pi}{9} =$

$75° =$

$54° =$

$\frac{2\pi}{15} =$

RADIAN APPLICATIONS WORKSHEET

1. Your analog clock shows the passage of 45 minutes.

   A. Find the radian angle through which the minute hand moves in that time.

1A. ____________

   B. Find the radian angle through which the hour hand moves in that time.

1B. ___________

2. The wheel of a bicycle is rotating at a rate of 90 revolutions per minute. Find the angular speed of a spoke of the wheel in radians per minute.

2. ____________

3. An electric hoist is used to lift a piece of equipment. The diameter of the drum on the hoist is 12 inches and the equipment must be raised 15 inches. Find the number of radians through which the drum must rotate?

3. _____________